PROBLEMS AND SOLUTIONS.

B. F. Finkel, Chairman of the Committee.

PROBLEMS FOR SOLUTION.

SPECIAL NOTICE. In proposing problems and in preparing solutions, contributors will please follow the form established by the Monthly, as indicated on the following pages.

In particular, a solution should be preceded by the number of the problem, the name and address of the proposer, the statement of the problem, and the name and address of the solver.

The solution should then be given with careful attention to legibility, accuracy, brevity without obscurity, paragraphing and spacing, having in mind the form in which it will appear on the printed page.

Please use paper of letter size, write on one side only, leaving ample margins, put one solution only on a single sheet and include only such matter as is intended for publication.

Drawings must be made *clearly* and *accurately* and an extra copy furnished on a *separate sheet* ready for the engraver.

Unless these directions are observed by contributors, solutions must be entirely rewritten by the committee or else rejected.

Selections for this department are made two months in advance of publication.

Please send all solutions direct to the chairman of the committee.

Managing Editor.

ALGEBRA.

416. Proposed by H. O. HANSON, East Elmhurst, N. J.

Find the nth term and the sum of n terms of the series obeying the relation $u_i = u_{i-1} + 2u_{i-2}$ in terms of n and the first two terms, u_1 and u_2, these two terms being arbitrary.

417. Proposed by ELMER SCHUYLER, Brooklyn, New York.

Solve $E(x^2) - E(3x) = 7$, where $E(m)$ is the largest integer in m. The value of x is to be in the form $4 + y/32$ where y is an integer.

GEOMETRY.

444. Proposed by S. A. COREY, Hiteman, Iowa.

Let $ABCDE$ be a pentagon, plane or gauche, with sides AB, BC, CD, DE, and EA. Bisect BC and DE in H and K respectively. Extend AB from B to B', and AE from E to E'. On AB' take sects AP and AV, and on AE' take sects AL and AT. Draw AD, AC, AH, AK, and DT. Let a, b, c, and d equal AL/AE, AT/AE, AV/AB, and AP/AB, respectively. Extend (or contract) AC from C to W, and AD from D to S, making $AW = a \times AC$ and $AS = d \times AD$. Draw LM and PN parallel to, and of the same currency as, AD and AC, respectively, and of lengths $c \times AD$ and $b \times AC$, respectively. Draw AM, AN, ST, and WV. Draw DQ and VX parallel to, and of the same currency as, CB and TS respectively. Prove that $2(ad + bc)(AK \times AH \times \cos KAH + KE \times HC \times \cos QDK) = AM \times AN \times \cos MAN + TS \times VW \times \cos WVX$.

Suggestion: Make use of the identity

$$[(w + x)a + (w - x)c][(z + y)d + (z - y)b] + [(w + x)b + (w - x)d][(z + y)c - (z - y)a] = 2(ad + bc)(wz + xy),$$

and solve with w, x, y, and z vectors, in a manner similar to that employed by the proposer in the solution of problems 377 and 383 in the May and October, 1911, numbers of the Monthly. The results in certain special cases are instructive, *e. g.*, when one or more of the sides, AC, CD, DE, are zero; when two or more of the sides lie in the same right line; when all the sides are tangent to a sphere, circle, ellipse, etc.; when the angles given in the above equation are multiples of some angle, etc.

445. Proposed by CLIFFORD N. MILLS, South Dakota State College.

Given the perimeter of a right triangle ABC and the perpendicular BD falling from the right angle B to the hypotenuse AC, to determine the sides of the triangle.

446. Proposed by S. G. BARTON, University of Pennsylvania.

Prove any one or more of the sixteen theorems, stated without proof, in the article in this issue of the MONTHLY (pages 182–184) on "Properties of the Normals to a Conic."

CALCULUS.

366. Proposed by I. A. BARNETT, University of Chicago.

Compute the definite integral $\int_a^b \sin^{-1} x dx$ where $0 \leqq a \leqq 1$ and $0 \leqq b \leqq 1$, by direct summation.

367. Proposed by C. N. SCHMALL, New York City.

Show that the volume enclosed by the surface $(x^2 + y^2 + z^2)^5 = (a^3x^2 + b^3y^2 + c^3z^2)^2$ is $\frac{4}{9}\pi(a^3 + b^3 + c^3)$.

MECHANICS.

295. Proposed by B. F. FINKEL, Drury College.

A homogeneous hollow cylinder, whose inner radius is half of its outer radius, rolls without slipping down a plane inclined at an angle α to the horizontal. Find its acceleration.

[From Prescott's *Mechanics of Particles and of Rigid Bodies.*]

NUMBER THEORY.

218. Proposed by ELIJAH SWIFT, Princeton, N. J.

If p is prime and > 3, show that $\sum_{a=1}^{a=p-1} 1/a^2 \equiv 0 \pmod{p}$.

219. Proposed by R. D. CARMICHAEL, Indiana University.

Determine whether it is possible for a polygon to have the number of its diagonals equal to a perfect fourth power.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

A solution of 399 by WM. CULLUM and a solution of 400 by LOUIS O'SHAUGHNESSY were received too late for credit in the May issue.

401. Proposed by R. D. CARMICHAEL, Indiana University.

Prove the validity of Borda's series:

$$\log (x + 2) = 2 \log (x + 1) - 2 \log (x - 1) + \log (x - 2) + 2\left[\frac{2}{x^3 - 3x} + \frac{1}{3}\left(\frac{2}{x^3 - 3x}\right)^3 + \frac{1}{5}\left(\frac{2}{x^3 - 3x}\right)^5 + \cdots\right].$$

SOLUTION BY A. M. HARDING, University of Arkansas.

We have

$$\log (x + 2) - 2 \log (x + 1) + 2 \log (x - 1) - \log (x - 2)$$

$$= \log \frac{(x - 1)^2(x + 2)}{(x + 1)^2(x - 2)} = \log \frac{x^3 - 3x + 2}{x^3 - 3x - 2} = \log \frac{1 + \dfrac{2}{x^3 - 3x}}{1 - \dfrac{2}{x^3 - 3x}}$$

$$= 2\left[\frac{2}{x^3 - 3x} + \frac{1}{3}\left(\frac{2}{x^3 - 3x}\right)^3 + \frac{1}{5}\left(\frac{2}{x^3 - 3x}\right)^5 + \cdots\right],$$

which proves the result.